Journal of Communications Technology, Electronics and Computer Science, Issue 11, 2017
ISSN 2457-905X

Application of Genetic Algorithm for Optimization Photonic Crystals

Alireza Rezaee
Assistant professor of Department of system and Mechatronics Engineering, Faculty of New sciences and technologies, University of Tehran, Tehran,IRAN
arrezaee@ut.ac.ir

***Abstract*— In this paper, great attention has been paid to photonic crystals due to their potential applications in ultra compact photonic integrated circuits. The goal of this work is applying a genetic algorithm to search for photonic crystals with large band gaps. The algorithm adopted in this work is based on introducing single-pixel perturbations to an initial seed with small relative band gaps. Each filling pattern of the grids is translated into a matrix of binary elements. By applying innovative crossover and mutation operators, the number of generations needed to achieve the fittest photonic crystal structure is considerably reduced. Besides, the required time to get optimal structure is significantly decreased. This is due to decreasing both the time needed to evaluate a band structure of each chromosome and generation number. Because of this efficient method, the required computation time of the fitness function is significantly decreased. The paper presents two optimized photonic crystal structures with almost 21% relative band gap.**

***Keywords*— genetic, crystal, optimization, photonic.**

I. INTRODUCTION

In recent years great attention has been paid to photonic crystals due to their potential applications in ultra compact photonic integrated circuits [1,2] such as optical waveguides [3,4], filters [5], lasers [6], and cavities [7]. Many of these applications take advantage of the photonic band gap imposed by the periodic structure of the propagating electromagnetic fields. So far, it has been discovered that the band gap of photonic crystals is controllable by the dielectric constant contrast of its constituent materials [8], the type of its lattice [9], the filling factor [1], the pattern used for the unit cell [10]. Since the appropriate performance of photonic crystals is dependent on their photonic band gap characteristics, it is of interest to design a photonic crystal with a maximum possible band gap. To this end, for a given dielectric material and a specific cell size, one should design the pattern of the unit cell with the goal of maximizing the relative photonic band gap.

One possible solution to the above problem has been based on the effect of dielectric veins and holes on the absolute band gap, which resulted in a hexagonal lattice of air holes [11]. An alternative solution has been suggested with regard to the fact that the photonic band gap size is strongly limited by the degenerate modes. Hence, one may propose approaches to reduce the structure symmetry in order to enhance the band gap. An anisotropic medium [12] or the insertion of small rods in the unit cell [13] allows a large absolute band gap for both square and hexagonal lattices.

One of the common methods to solve inverse problems is the genetic algorithm. The genetic algorithm originates from the evolution theory, and works by repeatedly selecting, varying, and replacing the best individuals in the next population [15]. Recently, by applying this algorithm to the photonic crystals, some 2D structures with large absolute band gaps for square [15] and triangular lattices [16] have been reported. The genetic algorithm has also been used to obtain large absolute band gaps for 2D anisotropic photonic crystals [17]. In addition, the genetic algorithm has been used to design structures with maximum band gaps for either TE or TM modes [18]. This algorithm is also a proper method for designing photonic devices with optimal performance [19].

The goal of this work is applying a genetic algorithm to search for photonic crystals with large band gaps. The algorithm is supported by a rigorous semi-analytical technique for band gap computation. The algorithm adopted in this work is based on introducing single-pixel perturbations to an initial seed with small relative band gaps. The variations of the initial seed obtained in the mentioned manner are selected as the initial population. Therefore, the structures are varied in such a way that they become closer to the structure showing the global maximum band gap.

II. METHODOLOGY

In this paper a 2D photonic crystal with square lattice consisting of two different dielectric materials is investigated. In order to apply the genetic algorithm, the unit cell is divided into many rectangular pixels represented by a binary value 1 (high dielectric constant) or 0 (low dielectric constant). Even for a unit cell with 100 pixels, there are 2100 various structures to search. Therefore, it is essential to exploit a fast and accurate method for evaluating the band structures of photonic crystals. For this purpose, we apply transmission line (TL) method to calculate effectively the band structure of a general 2D photonic crystal.

In [20] a detailed discussion of the TL formulation for two-dimensional dielectric photonic crystals is reported. It is shown how the full-wave analysis of photonic crystals can be reduced to the analysis of an equivalent multi-conductor transmission line. We begin with the following pseudo-Fourier series for the electric and magnetic field:

$$\boldsymbol{E}(\boldsymbol{r}) = \lim_{M,N\to\infty} \sum_{m=-M}^{M} \sum_{n=-N}^{N} \boldsymbol{E}_{mn}(z) e^{-j(\boldsymbol{k}+\boldsymbol{G}_{mn})\cdot\boldsymbol{r}} \tag{1}$$

$$\boldsymbol{H}(\boldsymbol{r}) = \lim_{M,N\to\infty} \sum_{m=-M}^{M} \sum_{n=-N}^{N} \boldsymbol{H}_{mn}(z) e^{-j(\boldsymbol{k}+\boldsymbol{G}_{mn})\cdot\boldsymbol{r}} \tag{2}$$

which should hold at every position vector $\boldsymbol{r} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$. In (1) and (2) $\boldsymbol{G}_{mn} = m\boldsymbol{b}_1 + n\boldsymbol{b}_2$ is a vector of the lattice reciprocal to the one defined by the vectors $\boldsymbol{L}_{mn} = m\boldsymbol{a}_1 + n\boldsymbol{a}_2$, while the primitive translation vectors $\boldsymbol{b}_i$ of the reciprocal lattice are given by

$$\begin{aligned} \boldsymbol{b}_1 &= \frac{2\pi}{S}(a_{2y}, -a_{2x}) \\ \boldsymbol{b}_2 &= \frac{2\pi}{S}(-a_{1y}, a_{1x}) \end{aligned} \tag{3}$$

where S is the area of the unit cell of the lattice, and $S = |\boldsymbol{a}_1 \times \boldsymbol{a}_2|$. The coefficients $\boldsymbol{E}_{mn}(z)$ and $\boldsymbol{H}_{mn}(z)$ along with $\boldsymbol{k} = k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}}$ denote the still unknown characteristics of a mode propagating parallel to the xy-plane. The unknown coefficients $\boldsymbol{E}_{mn}\cdot\hat{\boldsymbol{x}}$, $\boldsymbol{E}_{mn}\cdot\hat{\boldsymbol{y}}$, $\boldsymbol{H}_{mn}\cdot\hat{\boldsymbol{y}}$, and $\boldsymbol{H}_{mn}\cdot\hat{\boldsymbol{x}}$ arranged in the vectors $[\mathscr{E}_x]$, $[\mathscr{E}_y]$, $[\mathscr{H}_y]$, and $[\mathscr{H}_x]$, respectively, satisfy the following system of equations [20]:

$$\frac{d}{dz}\begin{pmatrix}[\mathscr{E}_x]\\ [\mathscr{E}_y]\end{pmatrix} = -j\omega[L]\begin{pmatrix}[\mathscr{H}_y]\\ -[\mathscr{H}_x]\end{pmatrix} \tag{4}$$

$$\frac{d}{dz}\begin{pmatrix}[\mathscr{H}_y]\\ -[\mathscr{H}_x]\end{pmatrix} = -j\omega[C]\begin{pmatrix}[\mathscr{E}_x]\\ [\mathscr{E}_y]\end{pmatrix} \tag{5}$$

the elements of the matrices $[L]$ and $[C]$ are in terms of a so-called refractive index matrix $[\mathscr{N}(z)]^2$ [20] which in its turn has the expansion coefficients of the dielectric constant $\varepsilon_r(\boldsymbol{r})$ as its elements. For the lattice configuration of this work, these are given by

$$\varepsilon_{mn}(z) = \frac{1}{S}\iint_S \varepsilon_r(\boldsymbol{r}) e^{j[(mb_{1x}+nb_{2x})x+(mb_{1y}+nb_{2y})y]} ds \tag{6}$$

in which S is the area of the unit cell of the two-dimensional crystal defined by the vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. According to [20], the modes of the photonic crystal are the frequencies in which matrices [L] (TM modes) or [C] (TE modes) have zero determinants. Therefore, using the TL method, evaluating the modes of a 2D photonic crystal can be transformed to calculating the eigenvalues of a matrix.

With the use of this method, the band structures of various structures are evaluated, and the best structures with larger fitness functions are selected. The fitness function of this work is assumed to be polarization independent relative band gap defined as the ratio of the bandwidth of the mid frequency of the band gap. Later, the fittest photonic crystals are exposed to mutation (randomly changing 0/1 binary values) and a 2D crossover (constructing the chromosomes of the two offsprings from the chromosomes of the parents). This procedure is repeated for many generations until the structure of the photonic crystal with the largest relative band gap is obtained.

Since the total number of possible structures of 2D structure photonic crystals enhances exponentially with the number of pixels, we assume that the primitive unit cell is invariant along the z axis and has the symmetry of being invariant under the mirror reflection with respect to the xz and yz planes. Therefore, the analysis of the photonic crystal structure can be reduced to the analysis of a quarter of a unit cell (Fig 1). Besides these symmetries, it is possible to assume that the unit cell has a mirror symmetry with respect to y=x plane. In this case, the analysis and search for the proper structure with the maximum relative band gap is reduced to one-eighth of the unit cell (Fig. 1).

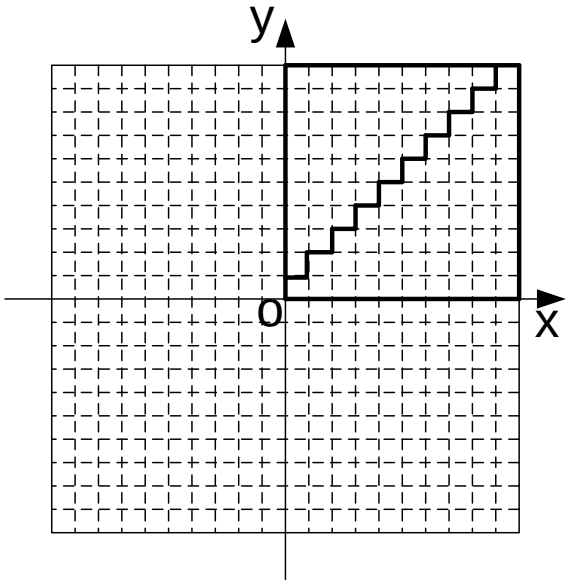


Fig. 1. Division of the unit cell of a 2D photonic crystal with square lattice. One of the unit cells has not the mirror symmetry with respect to the *y=x* plane and the other has the above mirror symmetry.

The steps toward obtaining photonic crystal structures with maximum band gap are given as the following. In the first step, we use a specific chromosome as the first chromosome of the initial population. The other individuals are generated by changing a single pixel of the first chromosome. In this manner, the initial population is constructed. For example, for 10×10 pixels in a nonsymmetrical structure with respect to the y=x plane, the initial population possesses 100 and in a symmetric structure only 55 2D chromosomes. Note that this procedure is suggested for improving the relative band gap for a predefined photonic crystal and also reducing the time required to find a more optimal one.

In the second step, the fitness function for each chromosome is computed. Using TL method based on matrix formulation, the time needed to obtain band structure is decreased drastically. In our work, computation of the fitness function for each chromosome is accomplished in 1.9 sec with the relative error of about 1%.

The third step contains three individual procedures for producing the next generation and again applying the second step. These three procedures are described as follows:

1. Selection: Suppose the unit cell is divided to 2M×2M pixels. From the previous generation or the initial population the M chromosomes with best fitness are selected. These chromosomes are also the parents for the new population.

2. Mutation: In this procedure, from each of selected chromosomes, M-2 new chromosomes are produced. To this end, one of the elements of the matrix of each selected chromosomes is randomly selected and subjected to a flip from 0 to 1 or vice versa.

3. Crossover: The last procedure consists of the random selection of two chromosomes of the parent population, first subjecting these two parents to an AND operator and second to an OR operator. If this is done for M times, 2M children are produced from the M parents.

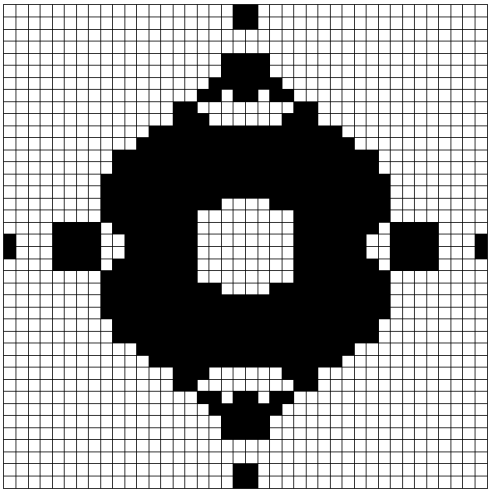
Therefore, each generation includes M(M+1) chromosomes. This process is repeated for many generations until the maximum fitness of a population does not vary after some generations. Hence, the structure of photonic crystal with the largest relative band gap is achieved. The property of this kind of selection is that the fitness of the population dose doesn't decrease in this newly created population. In the above algorithm one can optimize a structure for a specific M (for example 10) and take advantage of the obtained result to optimize the structure for a higher number of pixels (for example 20). The above idea is followed to obtain the following numerical results.

III. Numerical Results

As a numerical example, the two dielectric materials, constructing a photonic crystal structure are considered, air and GaAs with the relative dielectric constants 1 and 12.25 respectively. In this case at first, each chromosome consists of 10×10 pixels, then after optimization of the band gap, the number of pixels is increased to 20×20.

In the first example, the structure of a unit cell without a symmetry with respect to the y=x plane is optimized. The genetic algorithm was executed for 9 generations of 110 chromosomes with 10×10 pixels and 30 generations of 420 chromosomes with 20×20 pixels to achieve large band gaps for TE and TM polarizations. The best achieved structure with the largest fitness is shown in Fig. 2 (a), and its photonic band structure is shown in Fig. 2 (b). This structure has an absolute band gap of 0.92(2πc/a) at a midfrequency of 0.44(2πc/a) where c is the speed of light and a is the length of the unite cell. The above results correspond to 20.73% relative band gap.

As the second example, the structure of a unit cell which has a mirror symmetry with respect to the y=x plane is optimized. The genetic algorithm was executed for 13 generations of 110 chromosomes with 10×10 pixels and 16 generations of 420 chromosomes with 20×20 pixels to achieve the absolute large band gaps. Fig. 3(a) illustrates the achieved structure with the largest fitness, and its photonic band structure is shown in Fig. 3(b). This structure has an absolute band gap of 0.096(2πc/a) at a midfrequency of 0.45(2πc/a). The above results correspond to 21.13% relative band gap.



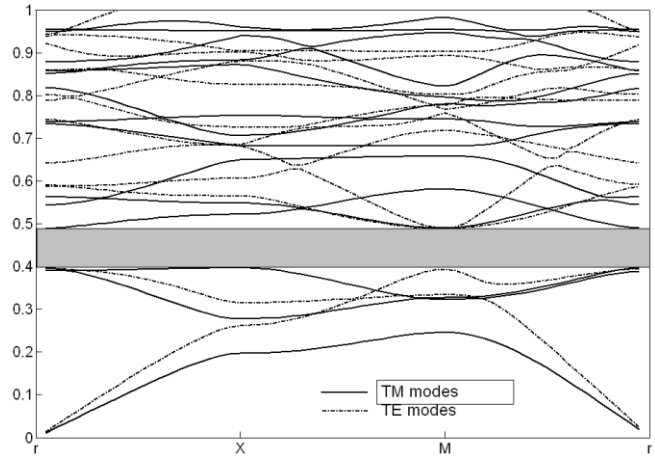


Fig. 2. (a) The best GaAs photonic crystal without mirror symmetry with respect to *y=x* plane. (b) The corresponding photonic band structures.

To show the performance of our method, we compare our results with those in [15]. In [15], after more than 104 generations with 100 populations (which contains 106 band gap evaluations), a structure with 20.1% relative band gap is obtained. However, in the present paper, a structure with 21% relative band gap is achieved with 1.3×104 band gap calculations for the first example and 8150 band gap calculations for the second example. It is obvious from these results that our method outperformed.

IV. Conclusions

The ability of a genetic algorithm to find novel photonic crystal structures with large relative band gaps is revealed. Starting with a photonic crystal structure that has very small band gap, one can achieve photonic crystals with larger band gap than the initial structure. By applying innovative crossover and mutation operators, the number of generations needed to achieve the fittest photonic crystal structure is considerably reduced. Besides, the required time to get optimal structure is significantly decreased. This is due to decreasing both the time needed to evaluate a band structure of each chromosome and generation number.

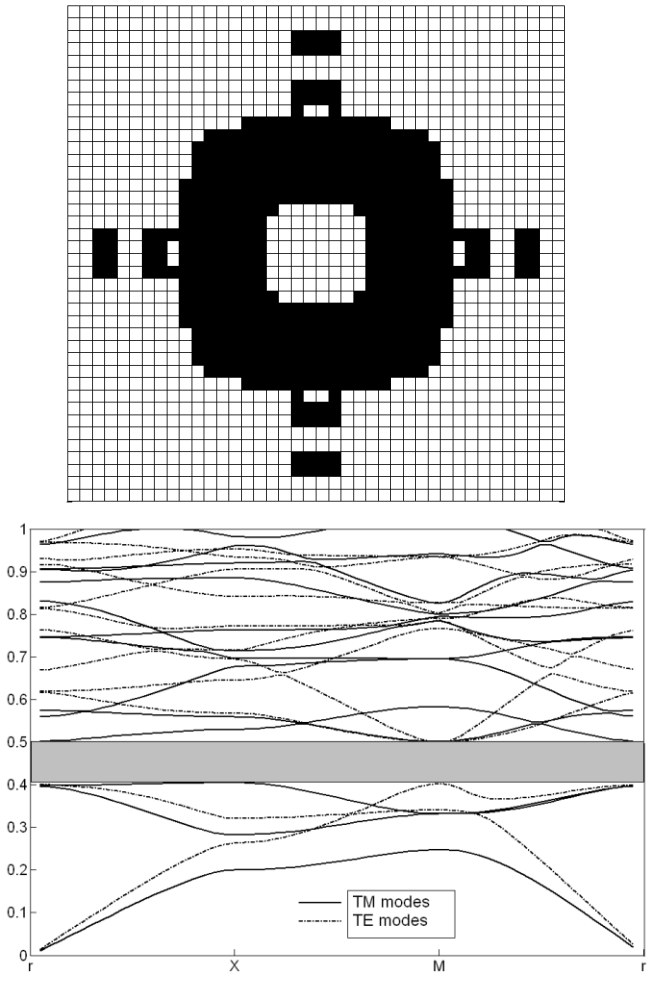


Fig. 3. (a) The best GaAs photonic crystal with mirror symmetry with respect to *y*=*x* plane. (b) The corresponding photonic band structures.

References

[1] J. D. Joannopoulos, R. D. Meade, and N. J. Winn, Photonic Crystals: Molding the Flow of Light Princeton, NJ, Sept. 1995.

[2] K. Busch, S. Lolkes, R. B. Wehrspohn, and H. Foll, Photonic Crystals: advances in design, fabrication, and characterization, WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim 2004.

[3] S. G. Johnson, S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, "Guided modes in photonic crystal slabs," Phys. Rev. B, vol. 60, pp. 5751–5758, 1999.

[4] S. G. Johnson, P. R. Villeneuve, S. Fan, and J. D. Joannopoulos, "Linear waveguides in photonic crystal slabs," Phys. Rev. B, vol. 62, pp. 8212–8222, 2000.

[5] M. Imdada, S. Noda, A. Chutinan, M. Mochizuk, and T. Tanaka, "Channel drop filter using a single defect in a 2-D photonic crystal slab waveguide," J. of Lightwave Tech., vol. 20, pp. 873-878, 2002.

[6] H. Y. Ryu, H. G. Park, and Y. H. Lee, "Two-dimensional photonic crystal semiconductor lasers: computational design, fabrication, and characterization," IEEE J. of Selected Topics in Quantum Electronics, vol. 8, pp. 891-908, 2002.

[7] R. D. Meade, A. Devenyi, J. D. Joannopoulos, O. L. Alerhand, D. A. Smith, and K. Kash, ''Novel applications of photonic bandgap materials: Low-loss bends and high-Q cavities,'' J. Appl. Phys. vol. 75, pp. 4753–4755, 1994.

[8] Z. Li, B. Gu, and G. Yang, "Large absolute band gap in 2D anisotropic photonic crystals," Phys. Rev. Lett., vol. 81, pp. 2574-2577, 1998.

[9] C. S. Kee, J. E. Kim, and H. Y. Park, " Absolute photonic band gap in a two-dimensional square lattice of square dielectric rods in air," Phys. Rev. E., vol. 56, pp. 6291-6293, 1997.

[10] C. M. Anderson and K. P. Giapis, "Larger two-dimensional photonic band gaps," Phys. Rev. Lett., vol. 77, pp. 2949-2952, 1996.

[11] R. D. Meade, K. D. Brommer, A. M. Rappe, and J. D. Joannopoulos, "Existence of a photonic band gap in two dimensions," Appl. Phys. Lett., vol. 61, pp. 495-497, 1992.

[12] Z. Li, J. Wang, and B. Gu, "Creation of partial band gaps in anisotropic photonic-band-gap structures," Phys. Rev. B., vol. 58, pp. 3721-3729, 1998.

[13] C. M. Anderson and K. P. Giapis, "Symmetry reduction in group 4mm photonic crystals," Phys. Rev. B., vol. 56, pp. 7313-7320, 1997.

[14] M. Mitchell, An introduction to genetic algorithms, MIT Press, 1996.

[15] L. Shen, Z. Ye, and S. He, "Design of two-dimensional photonic crystals with large absolute band gaps using a genetic algorithm," Phys. Rev. B., vol. 68, pp. 035109, 2003.

[16] C. Chen, A. Sharkawy, S. Shi, and D. W. Prather, "Band gap optimization of 2-dimensional photonic crystals," Proc. of SPIE, vol.5360, pp. 69-76, 2004.

[17] Z. Ye, L. Shen, and S. He, "Design for 2D anisotropic photonic crystal with large absolute band gaps by using a genetic algorithm," Phys. Rev. B., vol. 37, pp. 417-419, 2004.

[18] S. F. Preble, H. Lipson, M. Lipson, "Novel two-dimensional photonic crystals designed by evolutionary genetic algorithms," Proc. of SPIE, vol. 5597, pp. 118-128, 2004.

[19] A. Håkansson, J. S. Dehesa, and L. Sanchis, "Inverse design of photonic crystal devices," IEEE J. of Selected Areas in Communications, vol. 23, pp. 1365-1371, 2005.

[20] M. Shahabadi, S. Atakaramians, and N. Hojjat, "Transmission line formulation for the full-wave analysis of two-dimensional dielectric photonic crystals," IEEE Proc. Sci. Meas. Tech., vol.151, pp. 327-333, 2004.