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Performance analysis of PAPR in G-OFDM with different digital modulation methods

Sharmila Sengupta[1], Dr. B.K. Lande[2]
Associate Professor, Department of Computer Engineering, Vivekanand Education Society's Institute of Technology, Mumbai, 400074, India[1]
Professor, Department of Electronics and Telecommunication Engineering, Mumbai University, India[2]
sharmila.sengupta@ves.ac.in, bklande@gmail.com

***Abstract*—Multicarrier transmissions are proved to be robust against the frequency-selective features of a communication channel and orthogonal frequency division multiplexing (OFDM) is the promising multicarrier modulation technique used in several applications. The high power amplifier at the end of OFDM transmitter is usually operated near the saturation region for high output power efficiency leading to nonlinear distortion in the communication channels due to high peak to average power ratio (PAPR) of the input signals. In OFDM systems, error correction code is essential for transmitting signals at high signal rate. Coded-OFDM techniques were investigated earlier and was found to improve efficiency of wireless systems. In earlier paper, Goppa coded OFDM (GOFDM) was analyzed for its PAPR reduction capability as well as for forward error correction (FEC) for eight bit binary phase shift keying (BPSK) modulated data. But modulation techniques also play an important role in PAPR reduction and have been explained in this paper.**

Therefore other modulation techniques are tried with Goppa coded information symbols and their PAPR have been analyzed. As evident in the graphs, quadrature amplitude modulation (QAM) and quadrature phase shift keying (QPSK) shows better reduction in peak power of the symbols.

Keywords—OFDM, PAPR, FEC, Goppa codes, digital modulation, BPSK, QPSK, QAM.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing is a form of signal modulation used in many of the latest wireless and telecommunications standards, that divides and places a high data rate modulating stream onto many slow data rate modulated narrowband orthogonal subcarriers. It is less sensitive to frequency selective fading; multiple narrowband signals are affected individually as flat fading sub-channels. It is adopted in the Wi-Fi standards like 802.11a, 802.11n, 802.11ac and for the cellular telecommunications standard LTE / LTE-A, WiMAX and many more.

The OFDM receiver acts as a bank of demodulators, where the demodulated signal is integrated over the symbol period to regenerate the data from the received carrier. The same demodulator also demodulates other carriers but as the carrier spacing is equal to the reciprocal of the symbol period that is there are whole number of cycles in the symbol period and their contribution will sum to zero; in other words there is no interference contribution from other carriers. But non-linearity in the OFDM transmitting and receiving systems will cause interference between the carriers as a result of inter-modulation distortion and will impair the orthogonality of the transmission.

The RF final amplifier on the output of the OFDM transmitter is not able to handle the peaks of the transmitted signal against low average power leading to high peak to average PAPR ratio [1]. The amplifier cannot accommodate the large noise like amplitude variations and therefore cannot operate with a high efficiency level.

The deep fading of the channel degrades the overall performance affecting each subcarrier of an OFDM system. Therefore the efficiency of such systems is enhanced by applying different coding techniques otherwise reliable detection of the binary information would not be possible. The distribution of the data over many carriers will cause selective fading to cause errors in some received bits, By using an error-correcting code that is adding extra data bits at the transmitter, it is possible to correct many or all of the bits which were incorrectly received. Channel codes not only provide satisfactory FEC performance but also eliminates messages with highest peaks [2] by scrambling the data naturally and is proved to result in lesser PAPR variation.

II. LITERATURE SURVEY

The OFDM signal s(t) with N carriers over a symbol period of T is given by

$$s(t) = \sum_{n=0}^{N-1} d_n \, e^{\frac{j2\pi nt}{T}} \quad (1)$$

where d_n is the amplitude of the n^{th} subcarrier.

The peak to average power ratio for equally modulated carriers aligned in phase is given as

$$\mathrm{PAPR} = \frac{\max\{|s(t)^2|\}}{E\{|s(t)^2|\}} = \frac{N^2}{N} = N \quad (2)$$

Various methods of PAPR reduction are studied [3] which shows tradeoff of power efficiency with increase in data rate, bit error rate, transmission power, complexity etc. Most of these methods have in general used either of BPSK, QPSK or QAM modulation throughout their research.

Therefore effect of all these basic techniques in each PAPR reduction method is required to be investigated as constellation choice affects the peak power of transmitted signal. Even an all zero binary input data which tends to show high PAPR with BPSK shows considerable reduction in peak power with QAM technique. With higher order modulation methods, the number of carriers decreases; data rate increases and combined with low transmission power satisfies the requirement of an efficient system with a disadvantage of increased complexity in transceiver design.

III. Proposed System

A. Modulation of OFDM signal

In OFDM information symbols are mapped with the constellation of a digital modulation technique and then further modulated on to orthogonal carriers. BPSK, QPSK and QAM were tried on a 8 bit binary data and then Goppa coded 16 bit data. We know that $\mathrm{PAPR} = \frac{\text{Peak power}}{\text{Average power}} = \frac{\max[\mathrm{abs(power)}]}{\approx \mathrm{N}}$ (3)

For simplicity, explanation on 4 bit data is provided here.

The power of OFDM signal with binary data $d_n = d_1,d_2,d_3,d_4$ is $\mathrm{real}[s(t)s^*(t)] =$

$$[\sum_{n=1}^{4} d_n e^{j2\pi f_n t}][\sum_{n=1}^{4} d_n e^{*j2\pi f_n t}]$$

$$= d_1^2 + d_2^2 + d_3^2 + d_4^2 + 2d_1d_2 \cos(\theta_1 - \theta_2) + 2d_1d_3 \cos(\theta_1 - \theta_3) + 2d_1d_4 \cos(\theta_1 - \theta_4) + 2d_2d_3 \cos(\theta_2 - \theta_3) + 2d_2d_4 \cos(\theta_2 - \theta_4) + 2d_3d_4 \cos(\theta_3 - \theta_4)$$

(4)

where f_n is the frequency of the n^{th} subcarrier and $e^{j\theta} = e^{j2\pi f_n t}$

Considering $\theta_1 - \theta_2 = \alpha$ and $\theta_1 - \theta_3 = 2\alpha$ and so on, power =

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + 2d_1d_2 \cos\alpha + 2d_1d_3 \cos 2\alpha + 2d_1d_4 \cos 3\alpha + 2d_2d_3 \cos\alpha + 2d_2d_4 \cos 2\alpha + 2d_3d_4 \cos\alpha \qquad (5)$$

Therefore generalized equation for power of a N carrier OFDM signal

$$= \sum_{n=1}^{N} d_n^2 + 2\sum_{n,k=1;n\neq k}^{N} \cos(n-k), \qquad (6)$$

where $d_n d_k = \pm 1$.

The first part of the eq.(7) is a constant and depends on the number of carriers. Peak power will therefore depend on the second part of the equation and will be maximum with the maximum value of the summation of the product terms. Since average power = N, PAPR of OFDM symbols will depend on the above peak power.

A 4 bit data word d_1,d_2,d_3,d_4 = [0,0,0,1] after BPSK modulation in this example becomes = [-1,-1,-1,1], then its power would be $= 4 + 2\cos\alpha + 2\cos 2\alpha - 2\cos 3\alpha + 2\cos\alpha - 2\cos 2\alpha - 2\cos\alpha$. (7)

If 256 samples of the above signal is considered over 2π, the k^{th} sample will be at an angle $\alpha = \frac{2\pi k}{256}$.

$$\therefore power = 4 - 2\cos 3\alpha + 2\cos\alpha = 4 - 2\cos\frac{6\pi k}{256} + 2\cos\frac{2\pi k}{256} \qquad (8)$$

It can be derived that at α= 43 degrees that is at k = 1752 peak power = 7.07W from the above equation whereas for d_1,d_2,d_3,d_4 = [0,1,0,1] peak power is 16W.

As given in eq. (6), a generalised equation for QPSK-OFDM with m and k as the coefficients of its data can be similarly shown as

$$\mathrm{power} = \sum_{m=1}^{N/2} d_m d_m^* + \sum_{m,k=1:m\neq k}^{N/4} d_m d_{m+k}^* \cos(\theta_m - \theta_k) + \sum_{m,k=1:m\neq k}^{N/4} d_m^* d_{m+k} \cos(\theta_m - \theta_k) \qquad (9)$$

For QAM-OFDM with p and k as its coefficients, power =

$$\sum_{p=1}^{N/4} d_p d_p^* + \sum_{p,k=1:p\neq k}^{N/8} d_p d_{p+k}^* \cos(\theta_p - \theta_k) + \sum_{p,k=1:p\neq k}^{N/8} d_p^* d_{p+k} \cos(\theta_p - \theta_k) \qquad (10)$$

Comparing eq. (9) and (10) with eq. (6) it can be understood that all three techniques shows same results in their constant parts that is the first term of the respective equations. QPSK has N/4 cross modulation products and QAM has N/8 cross correlated terms and are therefore they are able to reduce more PAPR compared to BPSK and is quite evident even for an all zero sequence as shown in Fig. 13.

Thus peak power depends on the data symbols and indirectly would depend on their constellation mapping. To continue with the other modulation techniques [7] a general equation satisfying a mapping table as shown in table1 for reference is given below for $0 \leq t \leq T$.

$$\mathrm{Re}\{s(t)\} = \sum_{n=0}^{N-1} [a_n \cos(2\pi f_n t) + (b_n j\sin(2\pi f_n t)] \qquad (11)$$

TABLE1 VALUES OF COEFFICIENTS OF a_n AND b_n.

	BPSK	QPSK	16QAM
a_n	± 1	± 1	± 1, ± 3
b_n	0	± 1	± 1, ± 3

B. Coding of OFDM signal

In this paper a 8 bit data symbol is considered and then coded with (16,8,5) Goppa matrix G where G =

1 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0
0 1 1 1 0 1 0 1 0 1 0 0 0 0 0 0
1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1
0 1 1 1 1 0 1 0 0 0 0 1 0 0 0 0
1 0 1 1 1 1 0 0 0 0 0 0 1 0 0 0
1 0 1 0 1 1 0 1 0 0 0 0 0 0 1 0
1 0 0 1 1 1 1 0 0 0 0 0 0 1 0 0
0 1 1 0 0 0 1 1 0 0 1 0 0 0 0 0

Goppa codes, commonly referred as $(\mathrm{L}, \mathrm{g(z)})$ codes, is defined by the location set $L = \{\alpha_{1,\ldots,}\alpha_n\} \subseteq \mathrm{GF}(q^n)$ and a Goppa polynomial g(z) = $g_0 + g_1 z + \cdots + g_t z^t = \sum_{i=0}^{t} g_i z^i$

Here the Goppa polynomial used is $k(X) = X^4 + X^3 + 1$[4] which is the irreducible polynomial of degree 4 in the factorization of $X^{15}-1$.

The coded symbols are then modulated as per the constellation mapping of the modulation technique chosen and then multiplied by orthogonal carriers [5], to form an OFDM orthogonal burst for transmission. In fig.1, a similar method using QAM is described as a block diagram. The peak and average power of the OFDM burst is found out

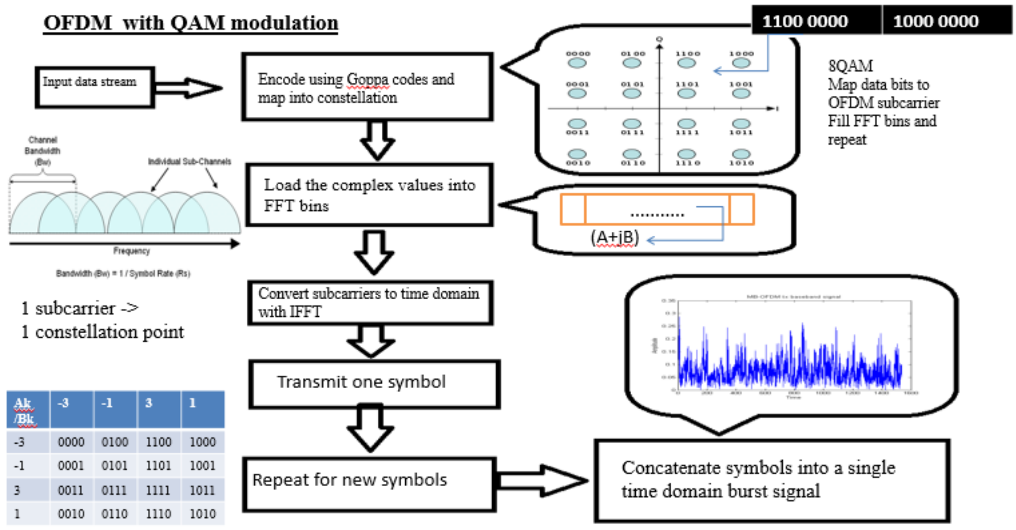

Fig. 1Process of QAM-OFDM burst generation using Goppa codes.

[Courtesy:http://rfmw.em.keysight.com/wireless/helpfiles/89600b/webhelp/subsystems/wlanofdm/content/ofdm_basicprinciplesoverview.htm]

through simulation in MATLAB and then PAPR in dB is calculated. QPSK-OFDM is explained in the following table where a two carrier OFDM burst for serial data 10 followed by 01 would be s(t) = (1-j) $\sin\omega_1 t$ + (-1+j) $\sin\omega_2 t$ where $\omega = 2\pi f$ and f_1 and f_2 are orthogonal carriers separated by $\nabla f = \frac{1}{T}$, and T is the symbol period.

TABLE 2 SYMBOLS MAPPED TO QPSK MODULATION TECHNIQUE

Time	$s_0(t)$	$s_1(t)$
t0	**1**	0
t1	**0**	1
t2	1	1
t3	0	0
t1+Δt	**1-j *sinω_1t**	-1+j *sinω_2t
t3+Δt	1-j *sinω_1t	1-j *sinω_2t

For 8 bit uncoded data converted to 16 bit Goppa codes with BPSK modulation, an OFDM system would require 8 and 16 carriers respectively for B-OFDM and B-GOFDM. Similarly Q-OFDM and Q-GOFDM would be a 4 and 8 carrier system respectively for QPSK. QAM-OFDM and QAM-GOFDM would be a 2 and 4 carrier system as per the constellation table in Fig. 1.

IV. Results

B-OFDM, Q-OFDM and QAM-OFDM are simulated in Matlab for 2^8=256 sequences in order and their results are compared with B-GOFDM, Q-GOFDM and QAM-GOFDM. An example of binary uncoded data=11100011 is taken for reference and its Goppa coded data is shown in Fig. 2 and Fig. 3 respectively.

The amplitude of the 256 codewords as shown in Fig. 4 is more in most of the cases in B-GOFDM than in B-OFDM; but PAPR depends on the peak power which may due to a certain cross modulation product of a subcarrier with other subcarriers. Therefore PAPR results in Fig. 5 does not sync with results of Fig. 4. Still Fig. 4 gives an idea that increasing number of carriers from 8 to 16 will certainly increase power of individual data.

Leaving aside the all zero code word and few other words, Goppa codes has certainly reduced PAPR of certain symbols; though it has increased it in some codewords; further methods like scrambling and interleaving [6] can be studied on them to reduce the peak power.

The average range of variation in 8 carrier B-OFDM is around 5 dB whereas in G-OFDM it is around 6 dB as seen in Fig. 5. It could be seen in Fig. 6 that the probability of PAPR > 8 dB is very less in G-OFDM; otherwise both B-OFDM and B-GOFDM almost follow each other between 2 to 7 dB. As mentioned earlier [4] certain codewords will give rise to high PAPR even after Goppa coding, but the advantage of using redundant bits for error correction and detection proves to be useful. This is why other modulation techniques were tried and results are explained hereafter.

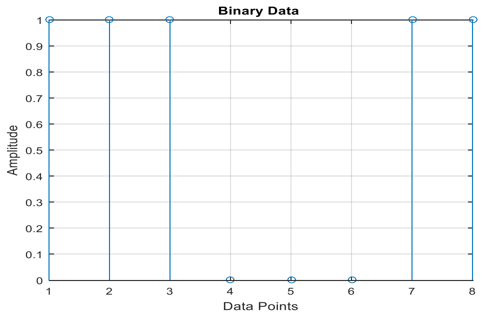

Fig. 2 Binary uncoded data=11100011

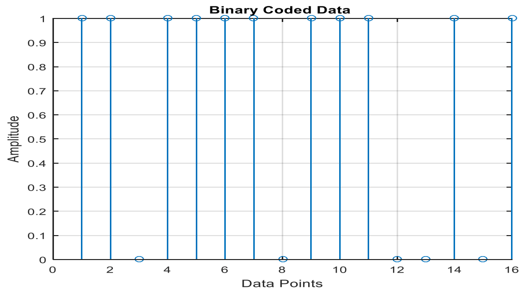

Fig. 3 Binary Goppa coded data

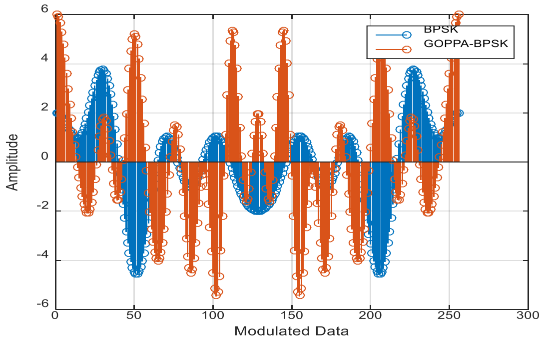

Fig. 4 Amplitude comparison of B-OFDM and B-GOFDM

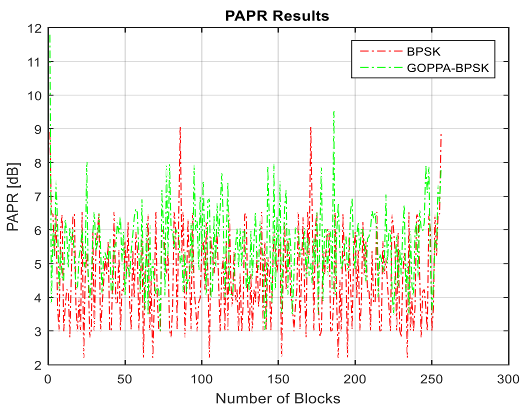

Fig. 5 PAPR comparison of B-OFDM and B-GOFDM

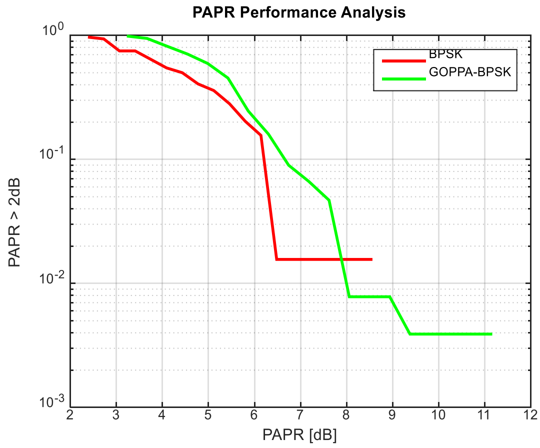

Fig. 6 Probability of PAPR> 2 dB in B-OFDM and B-GOFDM

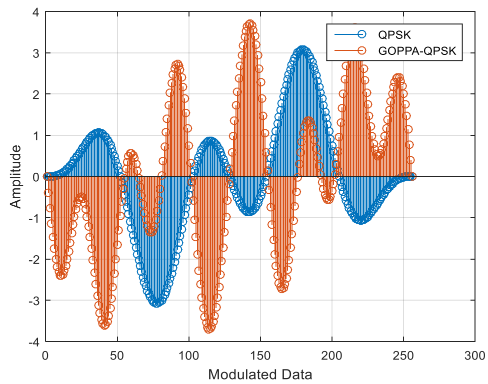

Fig. 7 Amplitude comparison of Q-OFDM and Q-GOFDM

Similarly individual power of each of the 256 codewords are shown for Q-OFDM and Q-GOFDM in Fig. 7 and fluctuations are much less compared to BPSK modulation. The average PAPR is around 5 dB in both the cases and leaving aside an all zero codeword and certain other symbols maximum PAPR in both the cases is around 7 dB as seen in Fig. 8. The probability of PAPR greater than 7 dB is very less from Fig. 9 but from 2 dB to 7 dB Q-GOFDM shows more variations than Q-OFDM. Still PAPR is 1 dB less in average compared to B-GOFDM with the advantage of less number of carriers and higher data rate.

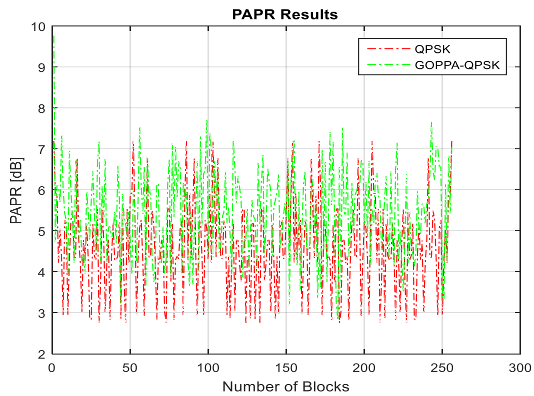

Fig. 8 PAPR comparison of Q-OFDM and Q-GOFDM

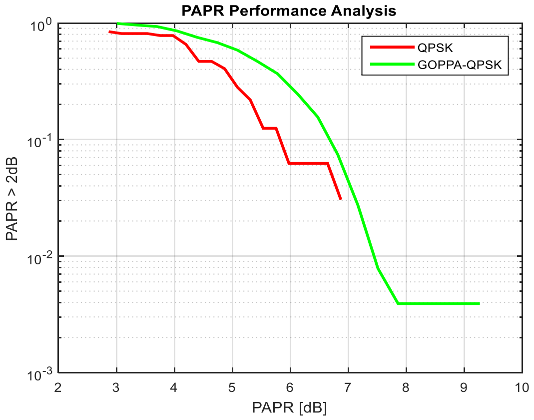

Fig. 9 Probability of PAPR> 2 dB in Q-OFDM and Q-GOFDM

QAM-OFDM shows much reduction in amplitude as seen in Fig.10. Average PAPR for QAM-GOFDM in Fig. 11 is 5 dB whereas for QAM-OFDM it is 3.5 dB which is quite less compared to other techniques. Only few codewords shows PAPR of 7 dB in QAM-GOFDM which can be attributed to the phase as well as amplitude changes in the signal constellation of QAM. There is almost no probability for PAPR greater than 4.8 dB in QAM-OFDM and very less probability of PAPR > 6 dB in QAM-GOFDM as evident from Fig. 12. In general QAM has an advantage of 1 dB reduction of PAPR than QPSK.

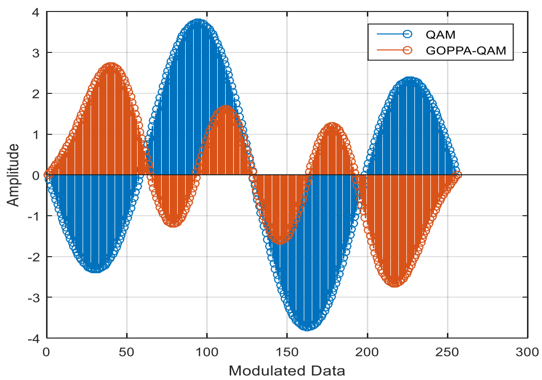

Fig.10 Amplitude comparison of QAM-OFDM and QAM-GOFDM

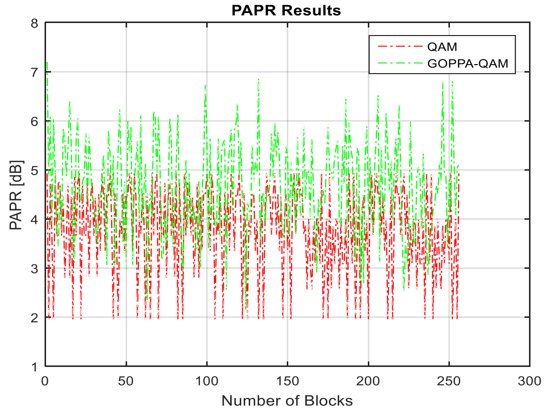

Fig. 11 PAPR comparison of QAM-OFDM and QAM-GOFDM

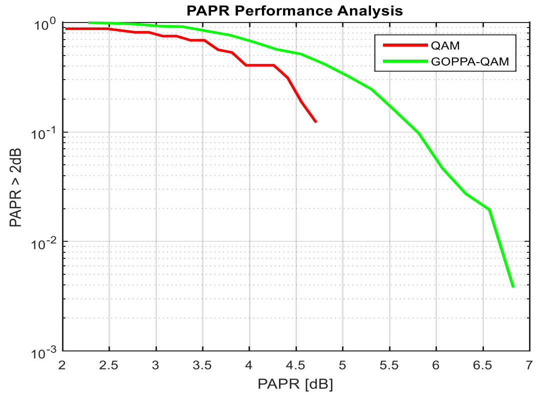

Fig. 12 Probability of PAPR> 2 dB in QAM-OFDM and QAM-GOFDM

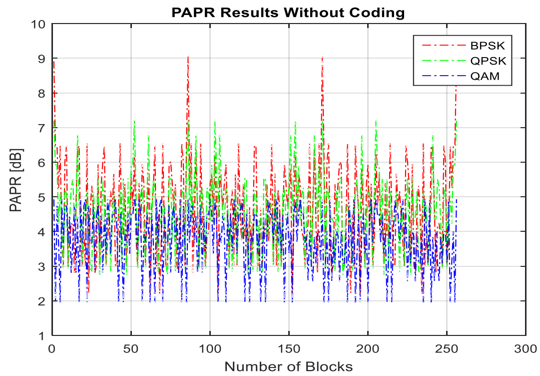

Fig. 13 PAPR comparison of B-OFDM, Q-OFDM and QAM –OFDM

Fig.13 and Fig.14 summarises the PAPR in all three modulation techniques for uncoded and coded data. QAM and QPSK stands better in PAPR reduction compared to BPSK in OFDM multicarrier modulation and verified for a certain data considered here as an example among 256 binary sequences in order.

The above mentioned results are also proved for 8 bit binary and its 16 bit Goppa coded data for random sequences as can be seen in Fig.15 and Fig.16.

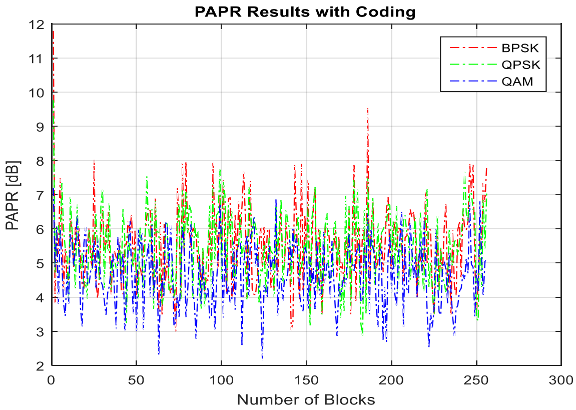

Fig.14 PAPR comparison of B-GOFDM, Q-GOFDM and QAM–GOFDM

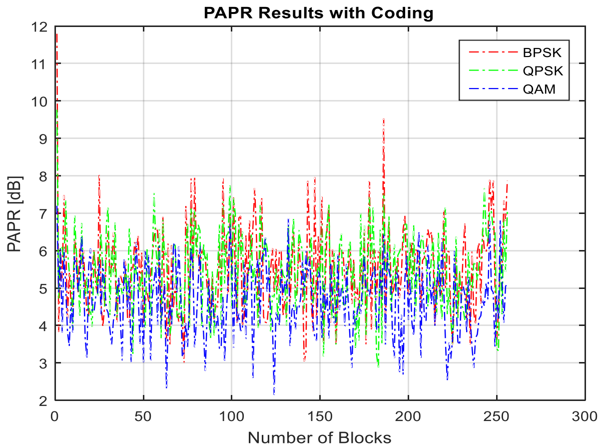

Fig.16 PAPR comparison of B-GOFDM ,Q-GOFDM and QAM-GOFDM for random binary 16 bit coded data

V. Conclusion

It was investigated earlier that OFDM systems perform well with extra bits added to it in terms of reduction of PAPR and also in performance in multi fading environments.

Therefore Goppa codes were tried earlier [4] and results showed improvement in PAPR reduction using BPSK as a modulation technique. It was understood through mathematical derivations that other digital modulation methods would certainly reduce peak power of OFDM symbols and simulation results verified that QAM and QPSK are better not only in terms of PAPR reduction but also in terms of less carrier requirement and therefore higher data rate. The complexity of the transceiver of QAM can be justified with high speed of transmitting symbols and FEC of the extra bits used in coding. Few code words whose PAPR exceeded a designated threshold has to be investigated further for reduction in peak power and can be tried with different decorrelation methods like scrambling and interleaving of data. The codewords which do not show

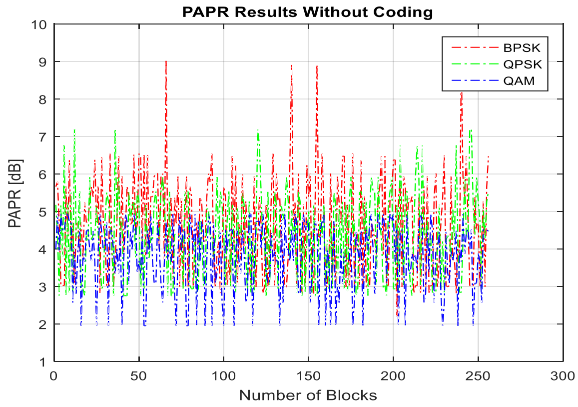

Fig.15 PAPR comparison of B-OFDM ,Q-OFDM and QAM-OFDM for random binary 8 bit data

peak power at the output of OFDM system can be transmitted without further correction and the redundant bits can be utilised for error detection and correction.

It is also proved here that in a codeword a certain harmonic of the code which are the cross correlation coefficients of the modulated data bits leads to high PAPR, that is where the correlation components sum up together in equal phase.

Therefore it is not necessary that all codewords will show high PAPR. Modulation techniques play a crucial role because of the mapping of data with different constellation structure which is investigated for BPSK, QPSK and QAM digital modulation methods in this paper.

VI. References

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